AS-2122

M.Sc. (Third Semester) Examination, 2013

PHYSICS

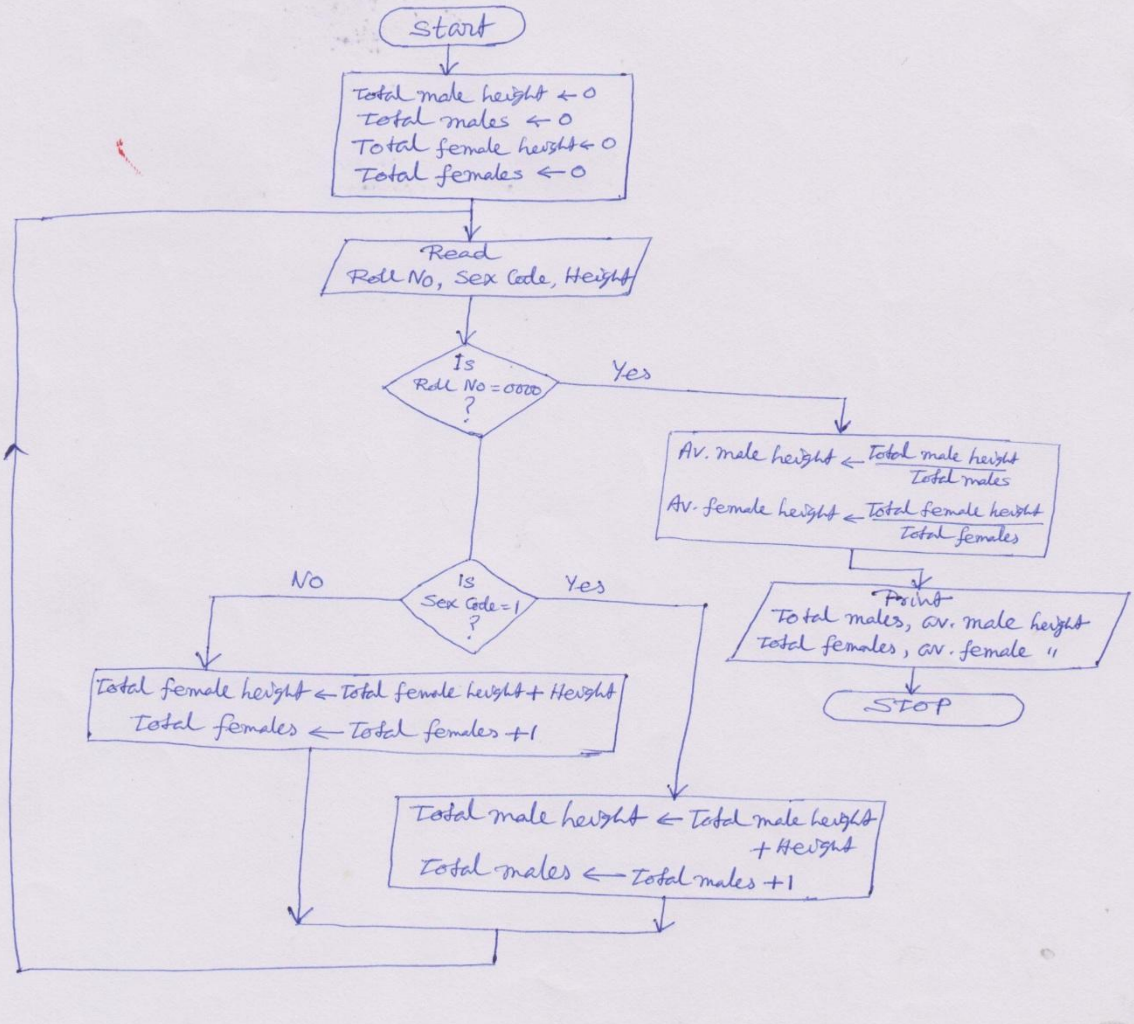
(Computer Programming and Numerical Analysis)

Section-A

1. (i) (d) (ii) (c) (iii) (a) (iv) (d) (v) (b)
   (vi) (c) (vii) (b) (viii) (b) (ix) (a) (x) (b)

Section-B

2. Draw a flow chart to find the average heights of males and females in a class separately.



3. Tabulate the following function using DO Loop

$$f(x) = (x^2+2x+3)/(x-4)$$

for $x = -6, -5, -4, -3, -2, -1, 0, 1, 2, 3$.

Ans

```
      DIMENSION  F(50)
      PRINT 90
 90   FORMAT (4X, 'X     F(X)'/)
      DO 80  I=-6,3
         X=FLOAT(I)
         F(X) = (X**2+2.0*X+3.0)/(X-4.0)
~~80~~    ~~CONTINUE~~
      PRINT 70, I, F(X)
 70   FORMAT(2X, I3, 3X, F6.2)
 80   CONTINUE
      STOP
      END
```

Output

X	F(X)
−6	−2.70
−5	−2.00
−4	−1.38
−3	−0.86
−2	−0.50
−1	−0.40
0	−0.75
1	−2.00
2	−5.50
3	−18.00

4. A university has the following rules for a student to qualify for degree with Physics as the main subject and Mathematics as the subsidiary subject. (a) He should get 50% or more in Physics and 40% or more in Mathematics.

(b) If he gets less than 50% in Physics he should get 50% or more in Mathematics. However, he should get at least 40% in Physics.

(c) If he gets less than 40% in Mathematics and 60% or more in Physics he is allowed to re-appear in an examination in Mathematics to qualify.

(d) In all other cases he is declared to have failed.

Prepare the decision table and write a program for that.

Ans A decision table for ~~Exam~~. Results

Physics Marks	≥50	≥40	≥60	Else
Mathematics Marks	≥40	≥50	<40	
Pass	✓	✓		
Repeat Mathematics			✓	
Fail				✓

```
C ** A PROGRAM FOR THE ABOVE DECISION TABLE **
      INTEGER PHYMAR, ROLLNO
      INFTY = 999
      DO 100 I=1, INFTY
      READ *, ROLLNO, PHYMAR, MATHMA
      IF (ROLLNO. EQ.0) GO TO 101
      IF((PHYMAR.GE. 50 .AND. MATHMA .GE. 40)
     1  .OR. (PHYMAR.GE.40 .AND.MATHMA .GE. 50)) THEN
         PRINT 10, ROLLNO, PHYMAR, MATHMA
10       FORMAT (10X, I6, 6X, I3, 6X, I3, 6X, 'PASSED')
      ELSEIF (PHYMAR .GE. 60 .AND. MATHMA .LT. 40) THEN
         PRINT 20, ROLLNO, PHYMAR, MATHMA
20       FORMAT(10X, I6, 6X, I3, 6X, I3, 6X, 'REPEAT MATH')
      ELSE
         PRINT 30, ROLLNO, PHYMAR, MATHMA
         FORMAT (10X, I6, 6X, I3, 6X, I3, 6X, 'FAIL')
      ENDIF
100   CONTINUE
101   STOP
      END
```

5. Apply Newton-Raphson method to determine a root of the equation

$$f(x) = \cos x - xe^x = 0$$

Newton-Raphson formula is given by

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}} = \phi(x_n)$$ and the convergence

condition is given by

$$|\phi'(x_n)| = \frac{f(x_n) f''(x_n)}{[f'(x_n)]^2} < 1$$

Here $f(x) = \cos x - xe^x$

$$\therefore f'(x) = -\sin x - e^x(x+1)$$

and $f''(x) = -\cos x - e^x(x+2)$

Taking $x = 0.5$,

$$f(x) = 5.3222 \times 10^{-2}$$

$$f'(x) = -2.9525 \qquad |\phi'(0.5)| < 1.$$

$$f''(x) = -3.0526 \times 10^{-2},$$

n	x_n	$f(x_n)$	$f'(x_n)$	$f(x_n)/f'(x_n)$	x_{n+1}
0	0.5	5.3222×10^{-2}	-2.9525	-1.8026×10^{-2}	0.51803
1	0.51803	-8.1942×10^{-4}	-3.0435	2.6858×10^{-4}	0.51776
2	0.51776	-1.1921×10^{-7}	-3.0421	3.9186×10^{-8}	0.51776

$\therefore$ The root of the equation $= 0.51776$.

6. Apply Gauss elimination method to solve the equations:

$$x+4y-z=-5 \quad \text{---} (1)$$
$$x+y-6z=-12 \quad \text{---} (2)$$
$$3x-y-z=4. \quad \text{---} (3)$$

First we have to eliminate x from eqn (2) & (3) and then eliminate y from eqn. (3) of the following three equation.

$$3x-y-z=4 \quad \text{---} (1)$$
$$x+4y-z=-5 \quad \text{---} (2)$$
$$x+y-6z=-12 \quad \text{---} (3)$$

Now multiplying eqn. (1) by -1 and (2) by 3 and adding we get $\quad 13y-2z=-19$

Similarly multiplying (1) by -1 and (3) by 3 and adding we get $\quad 4y-17z=-40$

So the set of equation becomes

$$\boxed{\begin{array}{ll} 3x-y-z=4 & \text{---} (1) \\ 13y-2z=-19 & \text{---} (4) \\ 4y-17z=-40 & \text{---} (4a) \end{array}}$$

Now we have to eliminate y from the last ~~two~~ eqns.

Multiplying (4) by $-4/13$ and add to (4a) we get

$$213z = 444$$
$$\text{or, } z = 148/71$$

Upper triangular form is therefore

$$\boxed{\begin{aligned} 3x - y - z &= 4 \quad &\text{--- (1)}\\ 13y - 2z &= -19 \quad &\text{--- (4)}\\ 71z &= 148 \quad &\text{--- (5)} \end{aligned}}$$

Back Substitution:

Putting (5) in (4) $\quad 13y - 2.\frac{148}{71} = -19$

$$\text{or, } y = -\frac{81}{71}$$

Now putting the value of y and z in (1) we get

$$3x = 4 - \frac{81}{71} + \frac{148}{71}$$

$$\text{or, } x = \frac{117}{71}.$$

$$\therefore x = \frac{117}{71} = 1.6479$$

$$y = -\frac{81}{71} = -1.1408$$

$$z = \frac{148}{71} = 2.0845$$

7. Use Lagrange's interpolation formula to find the value of y when $x=10$, if the following values of x and y are given:

x	5	6	9	11
y	12	13	14	16

Ans The Lagrange's interpolation formula is given by

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1)$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

where in this case

x	$x_0=5$	$x_1=6$	$x_2=9$	$x_3=11$
$y=f(x)$	$f(x_0)=12$	$f(x_1)=13$	$f(x_2)=14$	$f(x_3)=16$.

Using these values and putting $x=10$ in the above eqn.,

$$f(10) = \left(\frac{1}{6}\times 12\right) + \left(-\frac{1}{3}\times 13\right) + \left(\frac{5}{6}\times 14\right) + \left(\frac{1}{3}\times 16\right) = \frac{44}{3} = 14.67.$$

8. Using the Runge-Kutta method of fourth order, solve:

$$\frac{dy}{dx} = \frac{(y^2-x^2)}{(y^2+x^2)} \text{ with } y(0)=1 \text{ at } x=0.2, 0.4.$$

Ans The standard formulae for Runge-Kutta method of fourth order is given by

$$y_1 = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

where $k_1 = hf(x_0, y_0)$

$$k_2 = hf(x_0 + \tfrac{h}{2}, y_0 + \tfrac{k_1}{2})$$

$$k_3 = hf(x_0 + \tfrac{h}{2}, y_0 + \tfrac{k_2}{2})$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

In the present problem $y_0 = 1$ and $x_0 = 0$

$b = 0.4$ and $a = 0$.

Taking $h = 0.2$,

$$k_1 = hf(x_0, y_0) = 0.2 \times \frac{y_0^2 - x_0^2}{y_0^2 + x_0^2} = 0.2 \times \frac{1-0}{1+0}$$

$$= 0.2$$

$$k_2 = hf(x_0 + \tfrac{h}{2}, y_0 + \tfrac{k_1}{2})$$

$$= 0.2 \times \frac{(y_0 + k_1/2)^2 - (x_0 + h/2)^2}{(y_0 + k_1/2)^2 + (x_0 + h/2)^2} = 0.2 \times \frac{(1 + 0.2/2)^2 - (0 + 0.2/2)^2}{(1 + 0.2/2)^2 + (0 + 0.2/2)^2}$$

$$= 0.1967.$$

$$k_3 = 0.2 \times \frac{(y_0 + k_2/2)^2 - (x_0 + h/2)^2}{(y_0 + k_2/2)^2 + (x_0 + h/2)^2}$$

$$= 0.2 \times \frac{(1 + \frac{0.1967}{2})^2 - (0 + \frac{0.2}{2})^2}{(1 + \frac{0.1967}{2})^2 + (0 + \frac{0.2}{2})^2} = 0.1967$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

$$= 0.2 \times \frac{(y_0 + k_3)^2 - (x_0 + h)^2}{(y_0 + k_3)^2 + (x_0 + h)^2} = 0.2 \times \frac{(1 + 0.1967)^2 - (0 + 0.2)^2}{(1 + 0.1967)^2 + (0 + 0.2)^2} = 0.1891$$

$$\therefore y_1 = y(0.2) = 1 + \frac{1}{6}[0.2 + (2 \times 0.1967) + (2 \times 0.1967) + 0.1891] \simeq 1.1961$$

Now we have to find out the value of y at $x = 0.4$.

Runge-Kutta formula

$$y_2 = y_1 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

where

$$k_1 = hf(x_1, y_1)$$
$$k_2 = hf(x_1 + h/2,\ y_1 + k_1/2)$$
$$k_3 = hf(x_1 + h/2,\ y_1 + k_2/2)$$
$$k_4 = hf(x_1 + h,\ y_1 + k_3)$$

Here $y_1 = 1.196$; $h = 0.2$, $x_1 = 0.2$

$$\therefore k_1 = 0.2 \times \frac{(y_1)^2 - x_1^2}{(y_1)^2 + x_1^2} \simeq 0.189$$

$$k_2 = 0.2 \times \frac{(y_1 + k_1/2)^2 - (x_1 + h/2)^2}{(y_1 + k_1/2)^2 + (x_1 + h/2)^2}$$
$$= 0.2 \times \frac{(1.196 + \frac{0.189}{2})^2 - (0.2 + \frac{0.2}{2})^2}{(1.196 + \frac{0.189}{2})^2 + (0.2 + \frac{0.2}{2})^2} = 0.179$$

$$k_3 = 0.2 \times \frac{(y_1 + k_2/2)^2 - (x_1 + h/2)^2}{(y_1 + k_2/2)^2 + (x_1 + h/2)^2}$$
$$= 0.2 \times \frac{(1.196 + \frac{0.179}{2})^2 - (0.2 + \frac{0.2}{2})^2}{(1.196 + \frac{0.179}{2})^2 + (0.2 + \frac{0.2}{2})^2} = 0.179$$

$$k_4 = hf(x_1 + h,\ y_1 + k_3)$$
$$= 0.2 \times \frac{(y_1 + k_3)^2 - (x_1 + h)^2}{(y_1 + k_3)^2 + (x_1 + h)^2}$$
$$= 0.2 \times \frac{(1.196 + 0.179)^2 - (0.2 + 0.2)^2}{(1.196 + 0.179)^2 + (0.2 + 0.2)^2} = 0.169$$

$$\therefore y(0.4) = y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
$$= 1.196 + \frac{1}{6}[0.189 + (2 \times 0.179) + (2 \times 0.179) + 0.169]$$
$$= 1.375$$

Hence $y(0.2) = 1.196$

$y(0.4) = 1.375$